1. An impulse response is given by

$$H(u,v) = \exp\left(-\left[\frac{u^2}{150} + \frac{v^2}{150}\right]\right) + 1 - \exp\left(-\left[\frac{(u-50)^2}{150} + \frac{(v-50)^2}{150}\right]\right)$$

   (a) *What needs to be done to complete the characterization of the system?*

   Nothing needs to be done to complete the characterization of the system, since the impulse response is given. The impulse response is all that is needed to characterize a system.

   (b) *What is the function that the system performs? I.e., what filtering function does the system perform?*

   The impulse response is 2 Gaussian's, one centered at frequencies $u = v = 0$ and the other at $u = v = 50$, same spread, and opposite in magnitude. We can say that u and v are respective to the frequencies of the vertical and horizontal directions of an image.

   Since the first Gaussian (at $u = v = 0$) is positive AND unity is added to it, this is characteristic of a low frequency amplifier.

   Since the second Gaussian (at $u = v = 50$) is negative AND unity is added to it, at it's peak (at $u = v = 50$) the function equals 0. Therefore, this second Gaussian acts as a band-stop filter, removing frequencies within a specified spread around $u = v = 50$.

2. Image restoration model is given by

   - Spatial domain: $g(x,y) = f(x,y) * h(x,y) + \eta(x,y)$
   - Frequency domain: $G(u,v) = F(u,v) \times H(u,v) + N(u,v)$

   Cannon [1974] suggested a restoration filter $R(u,v)$ satisfying the condition,

$$\left|\hat{F}(u,v)\right|^2 = |R(u,v)|^2 \, |G(u,v)|^2$$

   (a) *Find $R(u,v)$ in terms of $|F(u,v)|^2$, $|H(u,v)|^2$, and $|N(u,v)|^2$*

   Since we can assume that the image and the noise are uncorrelated, we can say that

$$|G(u,v)|^2 = |F(u,v)|^2 \, |H(u,v)|^2 + |N(u,v)|^2$$

   We can also assume that the power spectrum of the restored image $\left|\hat{F}(u,v)\right|^2$ equals the power spectrum of the original image $|F(u,v)|^2$, so that

$$R(u,v) = \pm\sqrt{\frac{|F(u,v)|^2}{|F(u,v)|^2 \, |H(u,v)|^2 + |N(u,v)|^2}}$$

   (b) *Use the result in (a) to state a result in a form similar to the Wiener filter with the same terminologies used.*

   Since we know

$$\left|\hat{F}(u,v)\right|^2 = |R(u,v)|^2 \, |G(u,v)|^2$$

$$\left|\hat{F}(u,v)\right| = |R(u,v)| \, |G(u,v)|$$

Then we can substitute in for $R(u,v)$ to get

$$\left|\hat{F}(u,v)\right| = \left|\pm\sqrt{\frac{|F(u,v)|^2}{|F(u,v)|^2\,|H(u,v)|^2 + |N(u,v)|^2}}\right|\,|G(u,v)|$$

The power spectrum of the signal is given by $S_f = |F(u,v)|^2$ and the power spectrum of the noise is given by $S_n = |N(u,v)|^2$, so we get

$$\left|\hat{F}(u,v)\right| = \sqrt{\frac{S_f}{S_f\,|H(u,v)|^2 + S_n}}\,|G(u,v)|$$

$$\left|\hat{F}(u,v)\right| = \sqrt{\frac{1}{|H(u,v)|^2 + \frac{S_n}{S_f}}}\,|G(u,v)|$$

to finally get

$$\hat{F}(u,v) = \begin{cases} -\sqrt{\frac{1}{|H(u,v)|^2+\frac{S_n}{S_f}}}G(u,v), & G(u,v) < 0 \\ \sqrt{\frac{1}{|H(u,v)|^2+\frac{S_n}{S_f}}}G(u,v), & G(u,v) \geq 0 \end{cases}$$